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Aerodynamic Analysis Models for Vertical-Axis Wind Turbines

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This work details the progress made in the development of aerodynamic models for studying Vertical-Axis Wind Turbines (VAWT's) with particular emphasis on the prediction of aerodynamic loads and rotor performance as well as dynamic stall simulations. The paper describes current effort and some important findings using streamtube models, 3-D viscous model, stochastic wind model and numerical simulations of the flow around the turbine blades. Comparison of the analytical results with available experimental data have shown good agreement.

Key Words: *Wind turbines; Aerodynamics; Atmospheric turbulence; Dynamic stall; Navier-Stokes equations.*

Through advanced technology, wind turbine has become an important commercial option for large-scale power production. As a result, there are now more than 2000 megawatts of wind power in the world, most of them in the US and Denmark. Modern wind turbines can basically be classified as either of vertical-axis design, such as the Darrieus model, or as horizontal-axis variety like the traditional farm windpump. In both cases, the turbine's rotating blades extract kinetic energy from the wind to generate electricity or pump water. The vertical-axis wind turbine, such as Darrieus model with curved blades offers an advantageous alternative to the horizontal axis wind turbine due to its mechanical and structural simplicity of harnessing the wind energy. This simplicity, however, does not extend to the rotor's aerodynamic since the blade elements encounter their own wakes and those generated by other elements and operate in a dynamic stall regime (Brochier *et al.*, 1986). Added to this is the increasing awareness that atmospheric turbulence and fluctuating loads significantly affect the turbine output (Turyan *et al.*, 1987). The J.A. Bombardier Aeronautical Chair Group at École Poly-

technique de Montréal has conducted many research on the development of computer codes for studying Darrieus rotor aerodynamics (Paraschivoiu, 1988).

The objective of the computer programs is to determine aerodynamic forces and power output of the vertical-axis wind turbine of any geometry at a chosen rotational speed and ambient as well as turbulent wind. Three computer code variants based on the double-multiple streamtube model, stochastic wind and viscous flow field have been developed. The 3DVF viscous flow model based on Navier-Stokes equations analyses the Darrieus rotor in a steady incompressible laminar flow field by solving the Navier-Stokes equations in a cylindrical coordinates with the finite volume method where the conservation of mass and momentum are solved by using the primitive variables p , u , v , and w (Allet *et al.*, 1992). Since the ambient wind has been considered to be constant the predicted loads on the blades are identical for each rotor revolution. In order to take the fluctuating nature of the wind into account a 3-D stochastic model has been developed and incorporated first into the double-multiple streamtube model to analyse the effect of atmospheric turbulence on aerodynamic loads (Brahimi *et al.* 1992). Actually more work are underway to incorporate the stochastic wind into the 3DVF viscous model. For the dynamic stall simulation the computer

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codes developed use empirical model. Although the empirical dynamic stall models used predict well the aerodynamic loads and rotor performance, they are limited to the type of airfoil and motion used in the experiment from which they were derived. Thus, a new code based on the Navier-Stokes equations and uses the finite element method for simulating the dynamic stall around Darrieus wind turbine has been developed (Tchon *et al.*, 1993). Since 3-D simulation would be very expensive a 2-D simulation has been adopted. The model uses a non-inertial stream function-vorticity formulation (Ψ, ω) of the 2-D incompressible unsteady Navier-Stokes equations. The computer code was first validated for the flow around a rotating cylinder then, it was applied to simulate the flow around a NACA 0015 airfoil in Darrieus motion. The present paper presents some development of the streamtube models, the 3-D viscous model, the stochastic wind as well as the numerical simulation of dynamic stall.

MOMENTUM MODELS

Several aerodynamic prediction models currently exist for studying Darrieus wind turbines and a complete state of the art review including the appropriate references is given by Strickland (1986) and Paraschivoiu (1988). Generally, the main objective of all aerodynamic models is to evaluate the induced velocity field of the turbine since knowledge of this velocity field allows all the forces on the blade and the power generated by the turbine to be determined.

The first approach to analyze the flow field around vertical-axis wind turbine was developed by Templin (1974) who considered the rotor as an actuator disk enclosed in a simple streamtube where the induced velocity through the swept volume of the turbine is assumed to be constant. An extension of this method to the multiple-streamtube model was then developed by Strickland (1975) who considered the swept volume of the turbine as a series of adjacent streamtubes. Other aerodynamic methods for modeling the wind turbine are based on the vortex theory (Strickland, *et al.*, 1980). Basically two types of the vortex model have been used: the fixed-wake and the free-wake models. Although these models have the advantage to predict the aerodynamic loads and performance more exactly than the momentum models, they require a considerable amount of computer time. Paraschivoiu (1981) developed an analytical model (DMS) that considers a multiple-streamtube system divided into two parts where the upwind and downwind components of the induced velocities at each level of the

rotor are calculated by using the principle of two actuator disks in tandem. Three categories of computer codes have been developed (Fig. 1): *CARDAA* which uses two constant interference factors in the induced velocities calculated by a double iteration, *CARDAAV* code which considers the variation of the interference factors as a function of the azimuth and *CARDAAx* code which takes the streamtube expansion into account. These codes have been used at IREQ, Sandia National Laboratories, DAF Indal Co., IMST Marseilles and elsewhere.

For the upstream half-cycle of the rotor, the relative velocity and the local angle of attack as a function of tip speed ration "X" are given by:

$$W^2 = V^2 [(X - \sin\theta)^2 + \cos^2\theta \cos^2\delta] \quad (1)$$

$$\alpha = \arcsin \left[\frac{\cos\theta \cos\delta}{\sqrt{(X - \sin\theta)^2 + \cos^2\theta \cos^2\delta}} \right] \quad (2)$$

The normal and tangential forces coefficients are evaluated for each streamtube as a function of the blade position using the blade airfoil sectional force coefficients:

$$C_N = C_L \cos\alpha + C_D \sin\alpha \quad (3)$$

$$C_T = C_L \sin\alpha - C_D \cos\alpha \quad (4)$$

where the blade airfoil section lift and drag coefficients, C_L and C_D , are obtained by interpolating the available test data using both the local Reynolds number ($Re_b = W c/\nu$) and the local angle of attack.

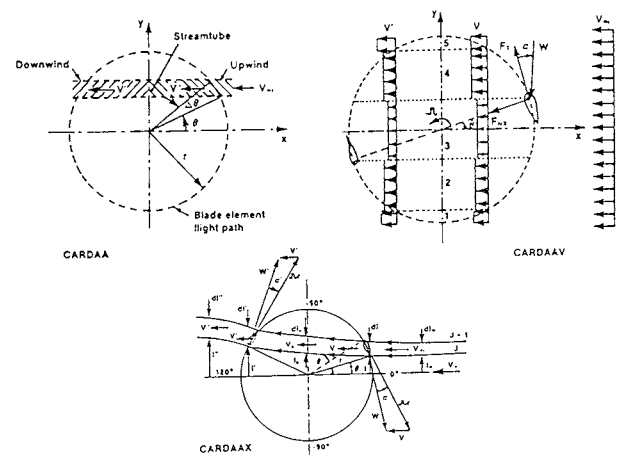


FIGURE 1 DMS model; *CARDAA*, *CARDAAV*, and *CARDAAx* computer codes.

STOCHASTIC WIND MODEL

The earlier aerodynamic models for studying Vertical Axis Wind Turbine (VAWT) are based on a constant incident wind conditions and are thus capable of predicting only periodic variations in the loads (Veers, 1984), (Malcolm, 1987), (Marchand *et al.*, 1987), (Strickland, 1987) and (Homicz, 1988). As a result, the predicted loads on the blade are identical for each rotor revolution and we have no information about the effect of turbulence on the rotor. Indeed the atmospheric turbulence as seen by rotating wind turbines has become an important factor for studying stochastic aerodynamic loads and turbulent flow effects have been identified as one of the major source of rotor blade fatigue life.

Continuing the development of the *DMS* model, a new code (*CARDAAS*) has been developed to predict loads on VAWT by taking the fluctuating nature of the wind into account. The velocity field of the wind is assumed to be a linear superposition of a steady or mean component and a fluctuating component. The main objective of the wind model is to simulate the turbulent velocity fluctuations. It includes both the streamwise and lateral component of the turbulent velocity. The one dimensional variations of this turbulent wind are introduced by creating time series of the wind velocity at a fixed point upwind the rotor and assuming that the wind speed is constant in a plane perpendicular to the mean wind direction.

The turbulent wind speed downstream of the fixed point is obtained by calculating a time delay in the time series. The decrease in the streamwise velocity as the flow passes through the rotor is taken into account by assuming a linear variation in the streamwise direction. The method used for the 3-D wind model is to simulate wind speed time series at several points in the plane perpendicular to the mean wind direction (Fig. 2). For

each point the time series is generated to represent the variation about the mean velocity in the longitudinal and vertical directions. The relative velocity and the local angle of attack are:

$$W^2 = [\Omega r - (V + u_f)\sin\theta - v_f\cos\theta]^2 + [(V + u_f)\cos\theta - v_f\sin\theta]^2 \cos^2\delta \quad (5)$$

$$\alpha = \arcsin [((V + u_f)\cos\theta - v_f\sin\theta)\cos\delta] / W \quad (6)$$

where Ω represents the turbine rotational speed, r the local rotor radius, V the induced velocity for each streamtube as a function of the azimuthal angle θ , u_f and v_f the fluctuating velocities and δ the meridional angle.

The fluctuation velocities due to the turbulent wind are represented by a Fourier time-series (Brahimi, 1992):

$$V_f^+ = \sigma^+ \sum_{j=1}^{N_p/2} [A_j^+ \sin(2\pi\eta_j^+ \tau^+) + B_j^+ \cos(2\pi\eta_j^+ \tau^+)] \quad (7)$$

where V_f^+ represents the normalized fluctuation velocities, $V_f^+ = V_f / V_{\infty i}$ ($V_f = u_f, v_f$), and σ^+ represents the normalized turbulence intensity, $\sigma^+ = \sigma / V_{\infty i}$. The unknown Fourier coefficients A_j^+ and B_j^+ are given in terms of the non dimensional spectral power density, Φ_i the dimensionless frequency band $\Delta\eta_j^+$ and a random phase angle ϕ_j by:

$$A_j^+ = (2\Phi_j\Delta\eta_j^+)^{1/2} \sin(\phi_j) \quad (8)$$

$$B_j^+ = (2\Phi_j\Delta\eta_j^+)^{1/2} \cos(\phi_j) \quad (9)$$

The fluctuation velocities are performed by using a Fast Fourier Transform, then the aerodynamic loads are evaluated for each streamtube as function of the blade positions using the total flow velocities.

THREE-DIMENSIONAL VISCOUS MODEL

Since the *DMS* codes do not take the viscous effects into account a computer code named 3DVF (Allet, 1993) has been developed. This code analyses the Darrieus rotor (Fig. 3) in a steady incompressible laminar flow field by solving the Navier-Stokes equations in a cylindrical coordinates using the finite volume method. The conservation of mass and momentum are solved using the primitive variables p , u , v , and w . The effect of the spinning blades is simulated by distributing source terms in the ring of control volumes that lie in the path of the turbine blades.

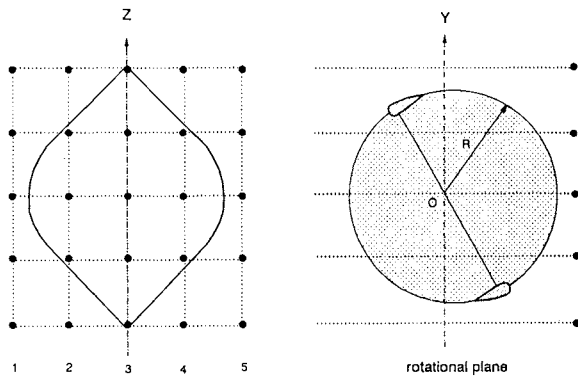


FIGURE 2 Schematic of 3-D wind simulation.

By using Fig. 3 the relative velocity \vec{V}_{rel} is calculated by the following equation:

$$\vec{V}_{rel} = V_n \vec{e}_n + \vec{V}'_\theta \vec{e}_\theta = (\vec{V}_{abs} \vec{e}_n) \vec{e}_n + (-\Omega r + \vec{V}_{abs} \vec{e}_\theta) \vec{e}_\theta \quad (10)$$

The only unknown parameters in the above equation (Eq. (10)) is the absolute velocity, \vec{V}_{abs} , which is computed by using the Navier-Stokes equations. The discretization method used to solve the governing equations is based on the finite volume method (FVM) proposed by Patankar (1980). For velocity-pressure coupled flows, a staggered grid system is known to give more realistic solutions and is adopted in the present study. The calculating method based on the control volume approach used here is the widely known "SIMPLER" algorithm. Details of the governing equations with the numerical procedure are given by Allet (1993). The motion of the Darrieus blades are time averaged and introduced through the source terms into the momentum equations. The source terms are valid for all the computational cells that lie in the path of the turbine.

$$S_r = NcpV_{rel} \frac{\Delta\theta}{2\pi} \cos\delta (C_{Ducos\delta} - C_L V'_\theta + C_D w \sin\delta) \quad (11)$$

$$S_\theta = NcpV_{rel} \frac{\Delta\theta}{2\pi} (C_D v + C_L V_n - C_D \Omega r) \quad (12)$$

$$S_z = NcpV_{rel} \frac{\Delta\theta}{2\pi} \sin\delta (C_D w \sin\delta - C_L V'_\theta + C_D u \cos\delta) \quad (13)$$

All forces and power computations are integrated for the grid points that lie in the path of the blades.

DYNAMIC STALL SIMULATION

Dynamic stall is an unsteady flow phenomenon which refers to the stalling behavior of an airfoil when the angle

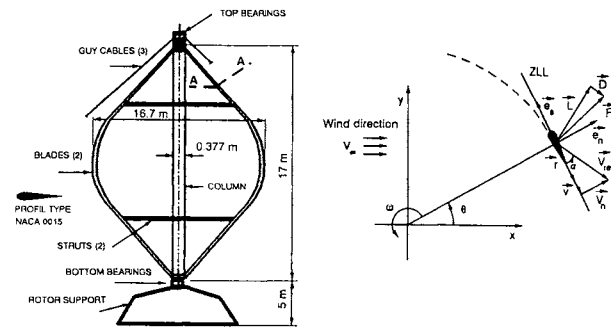


FIGURE 3 Angles, velocity vectors and forces for Sandia 17-m VAWT.

of attack is changing rapidly with time. It is characterized by dynamic delay of stall to angles significantly beyond the static stall angle and by massive recirculating regions moving downstream over the airfoil surface.

In the case of Darrieus wind turbine, when the operational speed approaches its maximum, all the blade sections exceed the static stall angle, the angle of attack changes rapidly and the whole blade operates under dynamic stall conditions. This increases the unsteady blade loads and structural fatigue (Ham, 1967, Philippe *et al.*, 1973, Gormont, 1973, McCroskey *et al.*, 1976 and McCrosky, 1981). Semi-empirical dynamic stall models have already been included in our computer codes based on DMS models as well as on the 3-D viscous model. These are namely the Gormont model (Gormont, 1973), MIT model and Indicial model (Paraschivoiu *et al.*, 1988, Proulx *et al.*, 1989). Although the dynamic stall models predict well the aerodynamic loads and performance on Darrieus wind turbine, they are limited to the type of airfoil and motion used in the experiment from which they were derived.

A new code for simulating the dynamic stall around Darrieus wind turbine has been developed, it is called "TKFLOW" (Tchon *et al.*, 1993). This code is based on the Navier-Stokes equations and uses the finite element method. Since 3-D simulation would be very expensive a 2-D simulation has been adopted. The model uses a non-inertial stream function-vorticity formulation (Ψ, ω) of the 2-D incompressible unsteady Navier-Stokes equations. The computer code was first validated for the flow around a rotating cylinder (Tchon *et al.*, 1990). Then, it was applied to simulate the flow around a NACA 0015 airfoil in Darrieus motion.

The vorticity transport equation and the stream function compatibility are given by:

$$\frac{\partial \omega}{\partial t} + \nabla \cdot [\omega \vec{u}] - \nabla v_e \omega + 2S_\omega \quad (14)$$

$$S_\omega = \left\{ \begin{array}{l} \frac{\partial v_e}{\partial x_1} \frac{\partial^2 \Psi}{\partial x_2^2} - \frac{\partial v_e}{\partial x_2} \frac{\partial^2 \Psi}{\partial x_1^2} \\ \frac{\partial v_e}{\partial x_2} \frac{\partial^2 \Psi}{\partial x_1^2} - \frac{\partial v_e}{\partial x_1} \frac{\partial^2 \Psi}{\partial x_2^2} \end{array} \right\}$$

The stream function compatibility equation is:

$$\nabla^2 \Psi + \omega = 0 \quad (15)$$

The effective viscosity is given by $v_e = \nu + \nu_t$ where ν_t represents the eddy viscosity. The computational mesh used in the TKFLOW is an hybrid one composed of a structured region of highly stretched quadrilateral elements in the vicinity of solid boundaries and an unstructured region of triangular elements elsewhere (Fig. 4).

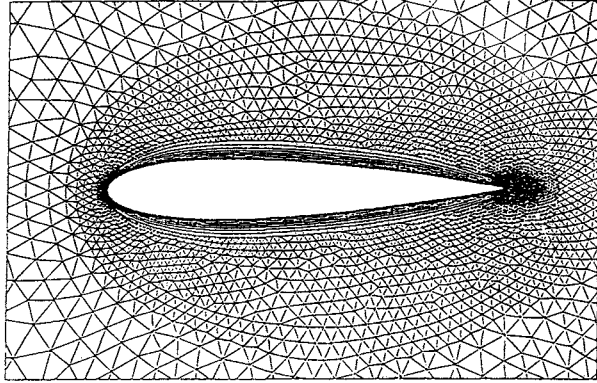


FIGURE 4 Computational mesh around NACA 0015 airfoil in Darrieus motion.

RESULTS AND DISCUSSION

The prediction of the performance coefficient vs tip speed ratio (TSR) for Sandia 17-m wind turbine using the DMS model is given by Fig. 5. Comparison with vortex model (VDART3) (Strickland *et al.*, 1980) and experimental data (Paraschivoiu, 1988) shows that the prediction by CARDAA code is well improved using CARDAAV and CARDAAX codes. Figs. 6 and 7 show the effect of atmospheric turbulence on the angle of attack and on the aerodynamic torque distribution. Unlike the periodic distribution predicted by CARDAAV, when turbulence is included the angle of attack distribution varies from one revolution to another. Furthermore, the ensemble-averaged aerodynamic torque distributions (Fig. 7) do not coincide with the periodic distribution. CARDAAS (1-D and 3-D) results predict well the experimental data given by Akins *et al.*, (1987).

The simulation of the dynamic stall hysteresis loop using 3DVF code with indicial model is given by Fig. 8.

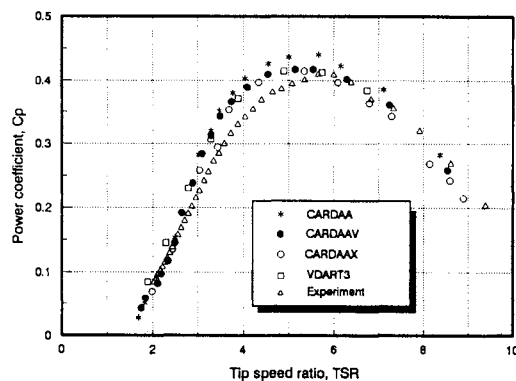


FIGURE 5 Performance coefficient vs tip speed ratio at RPM=50.6.

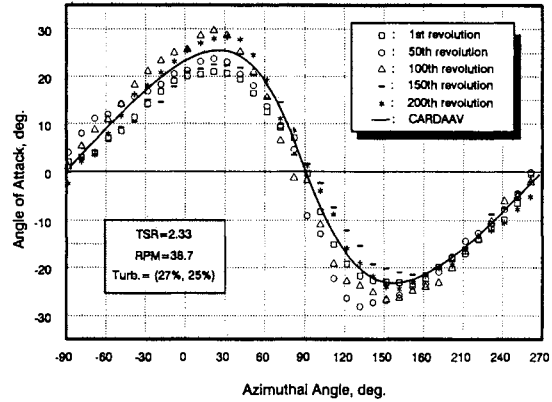


FIGURE 6 Angle of attack vs azimuthal angle at TSR=2.33 in turbulent wind.

Compared to CARDAAV results, predictions in the upwind and downwind regions of the turbine are well compared to experimental data (Akins, 1989). The performance predictions at different tip speed ratio (Fig. 9) is also well predicted using 3DVF code.

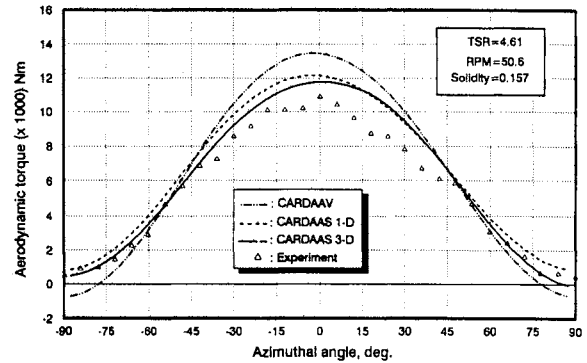


FIGURE 7 Aerodynamic torque at TSR=4.61 and turb.= (27%, 25%).

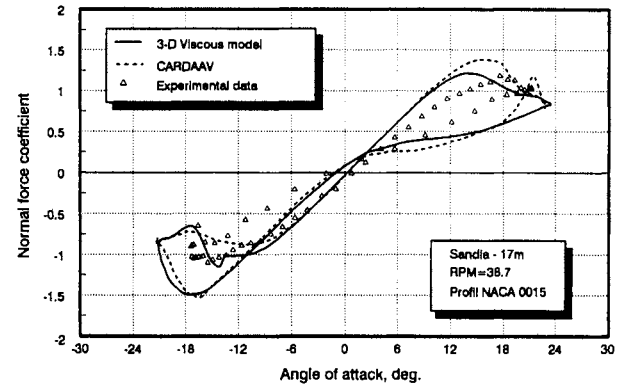


FIGURE 8 Normal force coefficient vs angle of attack at TSR=2.49.

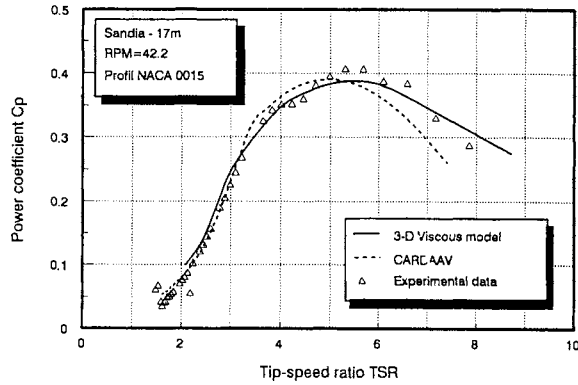


FIGURE 9 Power coefficient vs tip-speed ratio for Sandia 17-m.

In the above aerodynamic codes the model used for dynamic stall is based on semi-empirical methods. The simulation of the flow around an airfoil in Darrieus motion using Navier-Stokes solver is given by Fig. 10 in term of streamline evolution. Results using TKFLOW code can predict the region where the dynamic stall may occur.

CONCLUSION

Aerodynamic loads and performance of Darrieus wind turbine depend on the flowfield of the wind through the surface swept by the blades. The computer codes developed in this study represent a good tool for calculating the aerodynamic loads and performance of the vertical-axis wind turbines. Although the DMS models are still

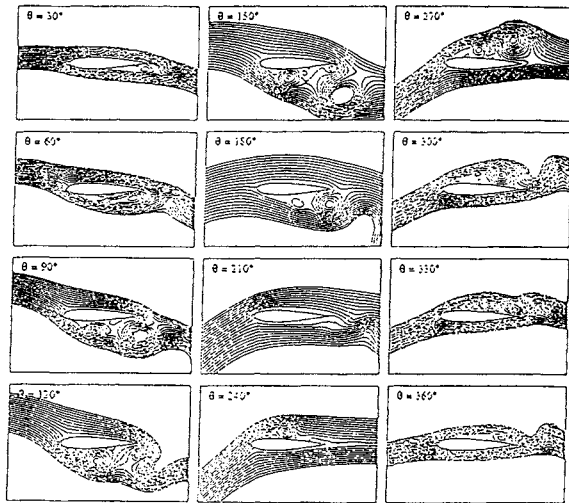


FIGURE 10 Computed streamlines around NACA 0015 airfoil in Darrieus motion.

being used in many places and are judged to be satisfactory because of its inexpensive approaches it is very important to include atmospheric turbulence into account especially for large wind turbines. In the case of wind turbine interferences such as wind farms the 3DVF seems to be more suitable since it can compute the flow velocity everywhere in the rotational plane as well as in its vicinity. The dynamic stall simulation using Navier-Stokes equations presents a good tool to predict the unsteady flow for Darrieus motion and more effort are underway to introduce a turbulence model in the present code.

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Nomenclature

A_j^*, B_j^*	:	Fourrier coefficients
c	:	blade chord, m
C_D	:	blade airfoil section drag coefficient
C_L	:	blade airfoil section lift coefficient
C_N	:	blade airfoil section normal-force coefficient
C_T	:	blade airfoil section tangential-force coefficient
N	:	number of blades
r	:	local rotor radius, m
Re_b	:	local Reynolds number
$S_{r,\theta,\omega}$:	source terms
TSR	:	tip speed ratio
u, v	:	components of the absolute velocity, m/s
u_f, v_f	:	fluctuations velocities, m/s
V_{abs}	:	absolute velocity, m/s
V_{∞}	:	local ambient wind, velocity m/s
W	:	relative inflow velocity, m/s
X	:	tip speed ratio
α	:	angle of attack, $deg.$
θ	:	azimuthal angle, $deg.$
δ	:	meridional angle, $deg.$
ν	:	cinematic viscosity, m^2/s
ν_e	:	effective viscosity, m^2/s
ν_t	:	eddy viscosity, m^2/s
Ω	:	turbine rotational speed, s^{-1}
ρ	:	density, kg/m^3
τ^+	:	dimensionless time
η^+	:	reduced frequency

References

- Akins, R. E., Berg, D. E., Cyrus, W. T., "Measurements and Calculations of Aerodynamic Torques for a Vertical Axis Wind Turbine", Sandia National Laboratories SAND86-2164, Albuquerque, NM, 1987.
- Akins, R. E., "Measurements of Surface Pressure on an Operating Vertical Axis Wind Turbine", Sandia National Laboratories SAND89-7051, Albuquerque, NM, 1989.
- Allet, A., "Modèle tridimensionnel pour le calcul aérodynamique des

- turbines à axe vertical", Thèse de doctorat, Département de Génie Mécanique, École Polytechnique de Montréal, 1993.
- Allet, A., Paraschivoiu, I., "Numerical Simulation of Three-Dimensional Flow Fields on Vertical-Axis Wind Turbines", The 4th International Symposium on Transport Phenomena and Dynamics of Rotating Machinery, (ISROMAC-4), Hawaii, pp. 214–223, 1992.
- Brahimi, M. T. and Paraschivoiu, I., "Stochastic Aerodynamic Model for Studying Darrieus Rotor in Turbulent Wind", (ISROMAC-3), Hemisphere Publishing Corporation, pp. 463–477, 1992.
- Brahimi, M. T., "Analyse aérodynamique du rotor Darrieus en présence d'un vent turbulent", Thèse de doctorat, Département de Génie Mécanique, École Polytechnique de Montréal, 1992.
- Brochier, G., Fraunié, P., Béguier, C. and Paraschivoiu, I., "Water Channel Experiments of Dynamic Stall on Darrieus Wind Turbine Blades", Journal of Propulsion and Power, Vol. 2, pp. 445–449, 1986.
- Gormont, R. E., "A Mathematical Model of Unsteady Aerodynamics and Radial Flow for Application to Helicopter Rotor", USAAMRDL, TR-72-67, 1973.
- Ham, N. D., "Aerodynamic Loading on a Two-Dimensional Airfoil During Dynamic Stall", Journal of the AIAA, Vol. 6, No 10, 1986.
- Homicz, G. F., "VAWT Stochastic Loads Produced by Atmospheric Turbulence", Presented at the 7th ASME Wind Energy Symposium, New Orleans, SED Vol 5, pp. 127–137, 1988.
- McCroskey, W. J., Carr, L. W., McAlister, K. W., "Dynamic Stall Experiments on Oscillating Airfoils", Journal of AIAA, Vol. 14, No 1, pp. 57–63, 1976.
- McCroskey, W. J., "The Phenomenon of Dynamic Stall", NASA-TM 81264, 1981.
- Malcolm, D. J., "Vertical Axis Wind Turbine Turbulent Wind Response Model", Final Report, Vol I and II, Indal Technologies Inc, 1987.
- Marchand, O., Brahimi, M. T., Paraschivoiu, I., "Stochastic Wind Effects on Vertical Axis Wind Turbines", Proceedings of the Canadian Wind Energy Conference, Calgary, pp. 273–301, 1987.
- Paraschivoiu, I., "Aerodynamic Loads and Performance of the Darrieus Rotor", Journal of Energy, Vol. 6, No. 6, pp. 406–412, 1981.
- Paraschivoiu, I., "Aerodynamic Models and Experiments for Studying Darrieus Wind Turbines", European Wind Energy Conference, Herning, Denmark, pp. 617–622, 1988.
- Paraschivoiu, I. and Allet, A., "Aerodynamic Analysis of the Darrieus Wind Turbines Including Dynamic-Stall Effects", Journal of Propulsion and Power, Vol. 4, No. 5, pp. 472–477, 1988.
- Patankar, S.V., "Numerical Heat Transfert and Fluid Flow", Hemisphere Publishing Corporation, New York, 1980.
- Philippe, J. J. and Sagner, M., "Calcul et Mesure des Forces Aérodynamiques sur un Profil Oscillant avec et sans Décrochage", Février 1973, AGARD, CP-111, 1973.
- Proulx, G., Paraschivoiu, I., "Methode Indicielle pour le Calcul du Décrochage Dynamique", Rapport Technique, Février 1989, Département de Génie Mécanique, Ecole Polytechnique de Montréal, EPM/RT-89/1, 1989.
- Strickland, J. H., "The Darrieus Turbine: A Performance Prediction Model Using Multiple Streamtubes", Sandia National Laboratory Report, SAND 75-041, 1975.
- Strickland, J. H., Webster, B.T. and Nguyen, T., "A Vortex Model of the Darrieus Turbine: An Analytical and Experimental Study", Sandia National Laboratories Report, SAND79-7058, 1980.
- Strickland, J. H., "A Review of Aerodynamic Analysis Methods for Vertical-Axis Wind Turbine", Proceeding of the 5th ASME Wind Energy Symposium, New Orleans, LA, pp. 7–17, 1986.
- Strickland, J. H., "VAWT Stochastic Wind Simulator", Sandia National Laboratories SAND87-0501, Albuquerque, NM, 1987.
- Tchon, K. F., and Paraschivoiu, I., "Finite Element Simulation of Unsteady Two-Dimensional Incompressible Viscous Flow", Canadian Aeronautics and Space Journal, Vol. 36, No. 4, pp. 236–246, 1990.
- Tchon, K. F., "Simulation numérique du décrochage dynamique sur un profil d'aile en mouvement de rotation Darrieus", Thèse de doctorat, Département de Génie Mécanique, École Polytechnique de Montréal, 1990.
- Tchon, K. F., and Paraschivoiu, I., "Navier-Stokes Simulation of the Flow Around an Airfoil in Darrieus Motion", the 16th Annual Energy-Sources Technology Conference and Exhibition, Houston, TX, Jan. 31-Feb. 4, pp. 11–18, 1993.
- Templin, R. J. "Aerodynamic Performance Theory for the NRC Vertical Axis Wind Turbine", NRC of Canada, Report LTR-LA-160, 1974.
- Turyan, K. J., Strickland, J. H., Berg, D. E., "Electric Power from Vertical-Axis Wind Turbines", Journal of Propulsion, Vol. 3, No. 6, pp. 481–493, 1987.
- Veers, P. S., "Modeling Stochastic Wind Loads on Vertical Axis Wind Turbines", Sandia National Laboratories SAND83-1909, Albuquerque, NM, 1984.

